Indian J.Sci.Res. 5 (1): 197-202, 2014
ISSN: 0976-2876 (Print)
ISSN: 2250-0138(Online)

DESIGNING A SUPPLY CHAIN NETWORK ON THE BASIS OF VARIATIONAL INEQUALITY

AREZU REZAZADEH[a1] AND MOHAMMAD ALIZADEHJAMAL[b]

[a]Msc in Mathematics, Ferdowsi University of Mashhad, Mashhad, Iran
[b]PhD Student in Mathematics, Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran

ABSTRACT

Increasing developments and changes in the field of production and business resulted in rising and development of concepts including supply chain and supply chain management. Supply change management from 190s onward developed increasingly among scientific societies, companies and industries and aimed at establishing cooperation among suppliers, lowering the costs, satisfying the needs of customers, increasing purchasing ability, profits and competitive advantages. The method applied in this study is Modified Image Method that is a common method to solve variational inequality. It assumes that K is a possible space and F is a function that acts in monotonous and Leap Sheets conditions. The method to solve inequality is proved and to apply it generally it was used on different issues with specified capacities. The result of proving the method of solution showed that it is an appropriate and reliable method to solve problems of chain supply management.

KEYWORDS: Variational Inequality, Supply Chain Network, Change Management

Recent developments in commerce and production lead scientists to offer the methods of supply chain management. Supply chain is a chain that covers the entire related activities to goods and converting materials from the beginning stage of preparing the raw materials to the final stage that is the time of delivering the goods to the customer. Beside the process of preparing materials there are two other processes that are the flow of information and the flow of financial resources. Supply chain management includes integrating activities of supply chain and related information flows to them by improving the relation of supply chain to reach at competitive advantages.

Most of the theorists believe that competitive principles cause the next decade to be based on competition between supply chains and emphasis on mixing the companies that are the member of one supply chain. Such issues caused offering of new approaches and attitudes about supply chain. The general purpose of designing supply chain is to provide an integrated network of facilities, firms, storages, distribution centers and customers to minimize the costs and to increase the profit of the firms with the most level of customer satisfaction. To reach at all these purposes an efficient supply network is needed. Such network has its specific features and limitations, for this reason the models of network design are various. In this study the researcher aims at offering the models of designing supply chain network using variational inequality to solve the problems of the network.

Is the previous method of solving the inequalities reliable to solve problems? And what is the best method of answering these problems? For example, how it is possible so solve the following problems:

1. One company increased the salary of workers from 250 \$ to 270\$ every month, what is the percentage of increasing prices that acquired from customers?
2. Calculate X:

A: $x - 10\sqrt{6} = 5\sqrt{6}$

B: $-x + 2\sqrt{3} = 5\sqrt{3}$

LITERATURE REVIEW

During the past several decades several models about supply chain were offered. These models use mathematical models like linear, non-linear, Fazi and statistical models. In the literature review we discuss some details about such models.

Linear Programming

The linear programming is an optimization issue that the purpose of its limit and function is linear. For example Chen and Wang (10) offered a leaner programming in the chain of supplying metals. Adil and Kanyalkar (8) offered a model of linear programming for designing production and dynamic distribution in one supply chain that has several products and factories. Oh and Karimi (5) offered a model of linear programming for production and distribution in multinational companies in one industrial section in the environment with several

[1]Corresponding author

factories in different time intervals. Other models of supply chain management were offered by Mula et al (2010).

Mixed Integer Linear programming (MILP)

Mixed integer linear programming is an optimization problem that its variables can be both integer and decimal and also their function can be linear. Most of the models in the supply chain are MILP. Mac Donald and Karimi(12) offered a model of liner integer for transference and production in chemical industries in the environment with several products and period of time. Goetchalckx (9) suggested the two models of mixed integer linear programming. Also Sakava (6) offered a model of MILA for production and distribution in a company of producing house materials in the Japan. Romeo (7) offered a mixed linear model to minimize production of natural gas in the Norway.

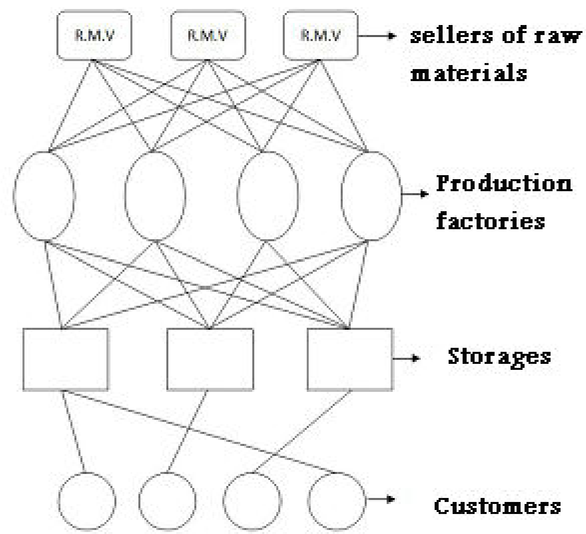
Nonlinear programming

Nonlinear programming is a program that limitations or the purpose function is nonlinear. Lababidi (17) offered a mixed integer nonlinear model to optimize the problem of supply chain in one petro-chemistry Company. Other models are documents in (11) that are Fazi, multi-purpose and statistical methods.

Samples of mathematical models in supply chain

Most of the supply chain models are (MILP). There are different models that here we refer to some of them. Jayarman (2001) offered a model of mixed integer linear to specify the place of production and distribution. This system needs two decisions; strategic, and operational. The purposes of distribution are affected by products of each factory, replacement of raw material from sellers to production and distribution factories to different places to customers.

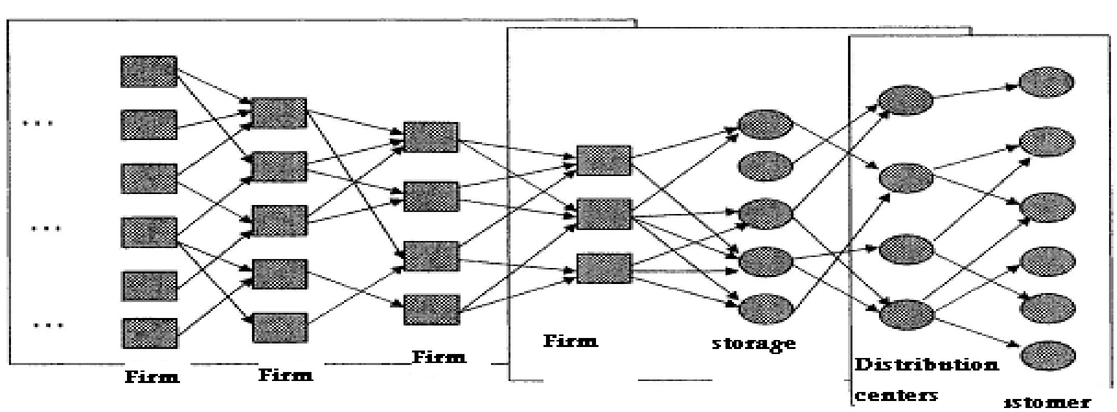
Figure 1: Jayarman Model (12)



The aim of this model is to minimize the stable costs and allocated variables to products based on limitations related to demand, production capacity, storage capacity and supplying of raw materials. This model uses three structures of costs that are: production cost, transferring cost and variable and stable price of distribution.

There is another model that was offered by Jang (2002). Hypotheses related to this model are as follow: the final capacity and performance prices related to centers of distribution, storages and factories are as follow. The purpose of this network is to minimize the performance prices and stable prices of the network.

Figure 2: Jang Model (11)



There are other models of supply chain the most important of them are the models offered by Amiri (2), Ko (16), Hinojosa et al (1), Jayarman et al (2), Listes (3) Selim et al (4).

Thought there is method of solving these inequalities, but variational inequality is a good model that can solve these models statistically.

METHODOLOGY AND EXAMPLES

The method that applied in this study is a model for solving variational inequalities. Simply variational inequality is used in solving nonlinear equations, completing problems, optimization and issues related to the stable point.

Definition: the problem of variational inequality $\mathrm{VI}(\mathrm{F},\mathrm{K})$ is calculation of the vector $\mathrm{X}^* \in \mathrm{K} \subseteq \mathrm{R}^n$ so that

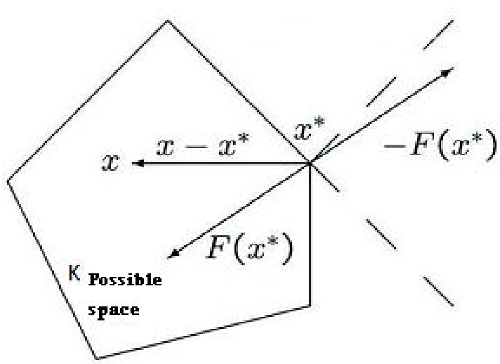
$$\mathrm{F}(\mathrm{X}^*)^{\mathrm{T}}(\mathrm{X}-\mathrm{X}^*) \geq 0 \qquad \forall \mathrm{X} \in \mathrm{K}$$

That can be shown as follow:

$$< \mathrm{F}(\mathrm{X}^*), \mathrm{X}-\mathrm{X}^* > \geq 0$$

K is a closed curve set $\mathrm{F}: \mathrm{R}^n \to \mathrm{R}^n$ a function and $< \cdots >$ represents internal coefficient.

Figure 3: Geometrical interpretation of variational inequality



The method of modified image is a repetitive method to solve variational inequality imagine that K^1 is the possible space and F is a function that adjusts to monotonous and leap sheets condition it means that,

$$\left(\mathrm{F}(\mathrm{X}^1)-\mathrm{F}(\mathrm{X}^2)\right)^{\mathrm{T}}(\mathrm{X}^1-\mathrm{X}^2) \geq 0$$

And there is one L so that,

$$\|\mathrm{F}(\mathrm{X}^1)-\mathrm{F}(\mathrm{X}^2)\|_2 \leq \mathrm{L}\|\mathrm{X}^1-\mathrm{X}^2\|_2 \qquad \forall\, \mathrm{X}^1, \mathrm{X}^2 \in K^1$$

Then the method of modified image is as follow:

$$X(t+1) = \left[X(t)-\gamma F\left(\bar{X}(t)\right)\right]^+$$

That,

$$\bar{X}(t) = \left[X(t)-\gamma F\left(\bar{X}(t)\right)\right]^+$$

γ is a positive scalar that is called the length of step. By $[X]^+$we mean the image of x on possible space of K^1.

If $\gamma \epsilon \left(0, \frac{1}{L}\right)$ then the stated method is convergent to an answer of variational inequality.

Argument: suppose that X* is an answer of variational inequality, then we have,

$$\begin{aligned} 0 \leq \left(F\left(\bar{X}(t)\right)-F(X^*)\right)^T(\bar{X}(t)-X^*) &= F\left(\bar{X}(t)\right)^T(\bar{X}(t)-X^*) \\ &\quad - F(X^*)^T(\bar{X}(t)-X^*) \end{aligned}$$

$$\begin{aligned} \leq F\left(\bar{X}(t)\right)^T(\bar{X}(t)-X^*) &= F\left(\bar{X}(t)\right)^T\left(\bar{X}(t)-X(t+1)\right) \\ &\quad + F\left(\bar{X}(t)\right)^T(X(t+1)-X^*) \end{aligned}$$

And finally

$$F\left(X(t)\right)^T\left(X^*-X(t+1)\right) \leq F\left(\bar{X}(t)\right)^T\left(\bar{X}(t)-\bar{X}(t+1)\right)$$

As X(t+1) is the image of $X(t)-\gamma F\left(\bar{X}(t)\right)$ on K^1 and $X^* \in K^1$ then from the upper equation we have,

$$\begin{aligned} \|X(t+1)-X^*\|_2^2 &\leq \left\|X(t)-\gamma F\left(\bar{X}(t)\right)-X^*\right\|_2^2 \\ &\quad - \left\|X(t)-\gamma F\left(\bar{X}(t)\right)-X(t+1)\right\|_2^2 \end{aligned}$$

$$= \|X(t)-X^*\|_2^2 - \|X(t)-X(t+1)\|_2^2 + 2\gamma F(\bar{X})^T\left(X^*-X(t+1)\right)$$

$$\leq \|X(t)-X^*\|_2^2 - \|X(t)-X(t+1)\|_2^2 - \|\bar{X}(t)-X(t+1)\|_2^2$$

$$-2\left(X(t)-\bar{X}(t)\right)^T\left(\bar{X}(t)-X(t+1)\right) + 2\gamma F\left(\bar{X}(t)\right)^T\left(X^*-X(t+1)\right)$$

And

$$\|X(t+1)-X^*\|_2^2 \leq \|\bar{X}(t)-X^*\|_2^2 - \|X(t)-\bar{X}(t)\|_2^2 - \|\bar{X}(t)-X(t+1)\|_2^2$$

$$+2\big(X(t+1)-\bar{X}(t)\big)^T\left(X(t)-\gamma F\big(\bar{X}(t)\big)\right)-\bar{X}(t)$$

As $\bar{X}(t)$ is the image of $X(t)-\gamma F\big(X(t)\big)$ on K[1] and $X(t+1)\epsilon K$ using leap sheets condition in the F finction we have,

$$\big(X(t+1)-\bar{X}(t)\big)^T\left(X(t)-\gamma F\big(\bar{X}(t)\big)-\bar{X}(t)\right)=$$

$$\big(X(t+1)-\bar{X}(t)\big)^T\left(X(t)-\gamma FX(t)-\bar{X}(t)\right)+$$

$$\gamma\big(X(t+1)-\bar{X}(t)\big)^T\left(F\left(X(t)-F\big(\bar{X}(t)\big)\right)\right)$$

$$\leq\gamma\big(X(t+1)-\bar{X}(t)\big)^T\left(F\left(X(t)-F\big(\bar{X}(t)\big)\right)\right)$$

$$\leq\gamma L\|X(t+1)-\bar{X}(t)\|_2\|X(t)-\bar{X}(t)\|_2$$

Now, using the two upper inequalities we have,

$$\|X(t+1)-X^*\|_2^2\leq\|X(t)-X^*\|_2^2-\|X(t)-\bar{X}(t)\|_2^2-\|\bar{X}(t)-X(t+1)\|_2^2$$

$$+2\gamma L\|X(t)-\bar{X}(t)\|_2.\|\bar{X}(t)-X(t+1)\|_2$$

$$=\|X(t)-X^*\|_2^2-(1-\gamma^2L^2)\|X(t)-\bar{X}(t)\|_2^2$$

$$-(\gamma L\|X(t)-\bar{X}(t)\|_2-\|\bar{X}(t)-X(t+1)\|_2)^2$$

$$\leq\|X(t)-X^*\|_2^2-(1-\gamma^2L^2)\|X(t)-\bar{X}(t)\|_2^2$$

Therefore, if $\epsilon\left(0,\frac{1}{L}\right)$, this method is convergent to an answer of variational inequality.

In addition, there are other designs of supply chain network with specified and obvious capacity and time limitation.

PROBLEM SOLUTION AND DISCUSSION

As mentioned the generalized image method is as follow:

$$X(t+1)=\left[X(t)-\gamma F\big(\bar{X}(t)\big)\right]^+$$

$$\bar{X}(t)=\left[X(t)-\gamma F\big(X(t)\big)\right]^+$$

In the article of Ananagorni the length of step for image method is considered 0.1 but in this article we analyze problems using length of step 0.01 for solving numerical examples and we compare them to answers by Ana Nagorni: in these examples every edge contains two cost that are:

$C_a(f_a)$= price of production, storage and transferring of product on the edge of a

$C_a(f_a)$ = the price of adding capacity on edge a

In all examples at first we calculate the process of ways that terminate in the center of K that we calculate it using the following equation:

$$X_{p_{w_k}}=\frac{d_{w_k}}{|P_{w_k}|}$$

$|P_{w_k}|$ is the number of ways that end to the center of demanding K. after calculating T_ps we can calculate f_as using the following equation,

$$f_a=\sum_{p\in p}X_p\,\delta_{ap}$$

u_a and λ_a are at first zero.

The condition of stopping for entire examples is as follow:

$$\|X(t+1)-X(t)\|\leq 10^{-5}$$

The processes for solving problems mentioned above, therefore, to prove our claim we analyze one example.

Ex: we imagine that there is a network as below that numbers on the edges represent the number of edges. This network is composed of three factories, two storages, two centers of distribution and three centers of demand. The rate of demands for these centers is as follow:

d_{R1}=45 d_{R2}=35 d_{R3}=5

having the upper assumptions and functions of costs documented in table 4-1 we solve this network. The generalized image method for this example reaches at answer after 900 repetitions. The rate of purpose function by this method is 15820.00 the value achieved by Ana Nagori in 497 repetition and length of step 0.1 is 16125.25. it was observed that by lowering the length of step, the answer became closer to the optimal value.

Figure 4: Supply chain management (22)

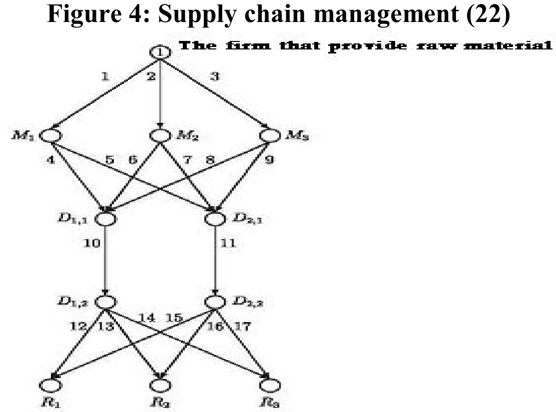


Table 1: Cost functions and answers

شماره یال ها					
1	$f_1^2 + 2f_1$	$0/5u_1^2 + u_1$	29/08	29/08	30/09
2	$0/5f_2^2 + f_2$	$2/5u_2^2 + u_2$	24/21	24/21	122/06
3	$0/5f_3^2 + f_3$	$u_3^2 + 2u_3$	31/63	31/63	65/27
4	$1/5f_4^2 + f_4$	$u_4^2 + u_4$	16/75	16/75	34/53
5	$f_5^2 + 3f_5$	$2/5u_5^2 + 2u_5$	12/23	12/23	63/12
6	$f_6^2 + 2f_6$	$0/5u_6^2 + u_6$	9/00	9/00	9/91
7	$0/5f_7^2 + 2f_7$	$0/5u_7^2 + u_7$	15/29	15/29	16/34
8	$0/5f_8^2 + f_8$	$1/5u_8^2 + u_8$	19/04	19/04	58/16
9	$f_9^2 + 5f_9$	$2u_9^2 + 3u_9$	12/49	12/49	52/97
10	$0/5f_{10}^2 + 2f_{10}$	$u_{10}^2 + 5u_{10}$	44/83	44/83	94/66
11	$f_{11}^2 + f_{11}$	$0/5u_{11}^2 + u_{11}$	40/17	40/17	14/18
12	$0/5f_{12}^2 + 2f_{12}$	$0/5u_{12}^2 + 3u_{12}$	25/36	25/36	25/38
13	$0/5f_{13}^2 + 5f_{13}$	$0/5u_{13}^2 + 3u_{13}$	17/98	17/98	18/94
14	$f_{14}^2 + 7f_{14}$	$2u_{14}^2 + 5u_{14}$	2/48	2/48	14/94
15	$f_{15}^2 + 2f_{15}$	$0/5u_{15}^2 + u_{15}$	20/65	20/65	21/62
16	$0/5f_{16}^2 + 3f_{16}$	$u_{16}^2 + u_{16}$	17/03	17/03	3506
17	$0/5f_{17}^2 + 2f_{17}$	$0/5u_{17}^2 + u_{17}$	2/50	2/50	3/50

It is possible to remove nonlinear costs on supply chain network and save extra money.

CONCLUSION AND FUTURE STUDIES

Supply chain network design was solved using the method of generalized image and using Matlab software and result was compared to Ana Nagorni method. It showed that this method better manages the cost and lowers the casts of products from the stage of production until reaching to customers. Previous studies represented some methods of transportation in the models of designing the network. In these models the nature of transportation has a binary nature, so that to transport products in one way every time it is possible to select one type of transportation. For example to carry products from one Production Company to the center of distribution in one period we can transfer then by land or by air and plane. Also some methods should be designed to facilitate the ways of transportation in emergency times.

REFERENCES

Hinojosa, Y., Kalcsics, J., Nickel, S., Puerto, J. and Velten, S., (2008), Dynamic supply chain design with inventory. Computers & Operations Research 35. 373-391.

Jayarman, V. and Ross, A., (2003). A simulated annealing methodology to distribution network design and management. European Journal of Operational Research 144, 629-645.

Listes, O., (2007). A generic stochastic model for supply and return network design. Computers & Operations Research 34, 417-442.

Selim, H. and Ozkarahan, I., (2008), A supply chain distribution network design model: an interactive fuzzy goal programming- based solution approach, International Journal of Advanced Manufacturing Technology 36, 401-418.

Oh, H.C. and Karimi, I.A., (2006). Global multiproduct production-distribution planning with duty drawbacks. AICHE Journal 52, 595-610.

Sakawa, M., Nishizaki, I. and Uemura, Y., (2001). Fuzzy programming and profit and cost allocation for a production and transportation problem. European Journal of Operational Research 131, 1-15.

Romo, F., Tomasgard, A., Hellemo, L, Fodstad, M., Eidesen, B.H. and Pedersen, B., (2009). Optimizing the Norwegian natural gas production and transport. Interfaces

Kanyalkar, A.P. and Adil, G.K., (2005). An integrated aggregate and detailed planning in a multi-site production environment using linear programming, International Journal of Production Research 43, 4431-4454.

Goetchalckx, M., Vidal, C.J. and Dogan, K., (2002). Modeling and design of global logistics systems: a review of integrated strategic and tactical models and design algorithms. European Journal of Operational Research 143, 1-18.

Chen, M. and Wang, W., (1997). A linear programming model for integrated steel production and distribution planning. International Journal of Operations and Production Management 17, 592-610.

Mula, J., Peidro, D., Diaz-Madronero, M. and Vicens, E., (2010). Mathematical programming models for supply chain production and transport Planning. European Journal of Operational Research 204, 377-390.

Mcdonald, C.M. and Karimi, I.A., (1997). Planning and scheduling of parallel semi-continious processes. Industrial and Engineering Chemistry Research 36, 2691-2700. 39(1), 45-56.